EE 553 Spring 2015 Homework 6

University of Washington

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Question 1

1.1 What are the principal characteristics of contracts?

- Contracts are agreements between two or more parties.
- All parties are obligated to meet contract requirements.
- Payment for contracts is done at delivery.
- Unconditional delivery.

1.2 Describe briefly the characteristics of spot contracts, forward contracts, futures contracts, and options contracts.

Section §2.4 in the book goes over this in good detail.

Spot Contracts
Spot contracts are those done on the day they are agreed upon — on the spot. Delivery of goods is done then too. Like a farmer's market, where the price is agreed upon and goods exchanged at the same time while the parties of the contract are present.

Forward Contracts
Forward contracts are those done with delivery of a commodity scheduled for a specific date of delivery and date of payment following delivery. Forward contracts can allow parties to lock in prices ahead of time by making a contract for a good before the good is ready to ship. Forward contracts help share the price risk between parties.

Futures Contracts
Futures contracts are similar to forward contracts except that the people buying and then selling the goods don't have anywhere to store them. They schedule a delivery date of the goods in the future and then sell those goods to someone else before the delivery date. This allows more traders to participate in the system. Like forward and spot contracts, delivery is unconditional. Since futures buyers cannot take delivery, they must sell any remaining goods they own futures to on the spot market prior to the delivery date.

Options Contracts
Options contracts are taking the marketing concept to the next level. They can be thought of as optional futures contracts. Essentially, someone with an option can choose to buy the goods at the rate of the option on or before the delivery date (call option) or sell the goods at the rate of the auction on or before the delivery date (put option). Europeon options must be used on the delivery date while American options can be done any time before the auction as well. When the option is used, the user of the option must pay the seller of the option a option fee.

1.3 Using a simple numerical example, explain the operation of a two–way contract for difference.

A two–way contract for difference is how two companies might bargain with one another to help share the risk of trading on the spot market.

Simple Example

Two farmers, John and Will, make a difference contract for quinoa with a price of \$7000.00 *per ton. John sells 8 tons of quinoa at* \$6500.00 *per ton on the open market while Will buys 8 tons of quinoa at* \$6500.00 *per ton on the open market. Because Will got his 8 tons for* \$6500.00 *per ton, he pays John* $8 * \$500 = \$4000.00$ *to make up the difference contract.*

In this example, Will effectively pays \$7000.00 *per ton and John does too. If John were a quinoa farmer and Will were using the quinoa for something, this would allow them to trade on the open market while not trading goods directly between themselves. So they could live across the country from one another and use a difference contract to make sure the sale worked out.*

Question 2

The little known country of Syldavia is considering the introductino of competition in electricity supply. Government economists estimate that the generation cost for electrical energy fits the following formula:

$$\boldsymbol{C = 1.5Q^2 + 100Q}$$

Where:

- **Q is the quantity produced [MWh]**
- **C is the total cost [$]**

They also estimate that the demand curve obeys the following relation for hour of peak demand:

$$\boldsymbol{P = -20D + 4000}$$

Where:

- **D is the demand [MWh]**
- **P is the price [$/MWh]**

Note: Numerical calculations should be carried out with two decimal places.

Note that I maintained most of my answers in rational form throughout the solution process so as to minimize the effects of rounding on my results until the very end.

2.1 Draw the supply curve.

Taking the derivative of the given generation cost function, we get the supply curve:

$$\begin{aligned} C &= 1.5Q^2 + 100Q \\ \text{supply curve: } \frac{dC}{dQ} = C' &= 3Q + 100 \end{aligned}$$

Instead of plotting just the supply, I have plotted both the supply and demand curves also showing the equilibrium point in Figure 1.

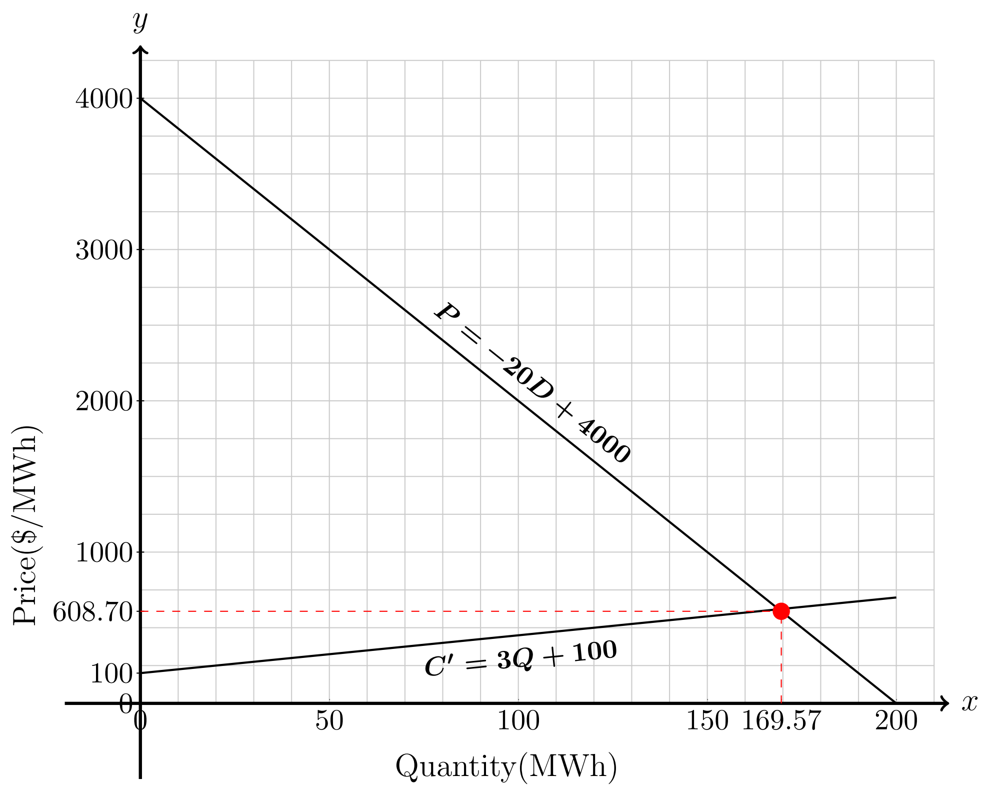

Figure 1: A plot of the supply and demand curves. The supply curve is labeled $C' = 3Q + 100$ while the demand curve is labeled $P = -20D + 4000$. The red point is the equilibrium point with the [$/MWh] and [MWh] shown in red dashed lines.

2.2 Calculate the price and quantity at which the market would clear for the hour of peak demand.

The market clearing price for the hour of peak demand is the solution for the intersection of the demand curve and the supply curve. The equation terms make this confusing. Since Q and D are both in MWh, we can solve with the equations as follows:

$$C' = 3Q + 100 \qquad\qquad P = -20D + 4000$$

$$\begin{aligned} 3Q + 100 &= -20D + 4000 \\ 23Q &= 3900 \\ \therefore Q = D &= \frac{3900}{23} \approx 169.56MWh \end{aligned}$$

At a demand and supply of $169.56MWh$, the price is calculated:

$$3\left(\frac{3900}{23}\right) + 100 = -20\left(\frac{3900}{23}\right) + 4000 = \frac{14000}{23} \approx 608.70\$/MWh$$

The market would clear at a price of $\$608.70/MWh$ and a quantity of $169.56MWh$.

2.3 Calculate the elasticity of demand at equilibrium.

From §2.2.1.4, *price elasticity of demand* is defined as "the ratio of the relative change in demand to the relative change in price."[1]

$$\varepsilon = \frac{\frac{dq}{q}}{\frac{d\pi}{\pi}} = \frac{\pi}{q}\frac{dq}{d\pi}$$

where $q = D(\pi)$ is the demand function and $\pi = D^{-1}(q)$ is the price function with q being quantity and π being the price. In this problem, we have the price function $P = -20D + 4000$ where P is the price and D the quantity. In order to find $\frac{dq}{d\pi}$, we must solve for our demand function:

$$\begin{aligned} P &= -20D + 4000 \\ P + 20D &= 4000 \\ 20D &= -P + 4000 \\ \therefore D &= -\frac{1}{20}P + 200 \end{aligned}$$

Now we cal solve for the *price elasticity of demand* at the equilibrium:

$$\varepsilon = \frac{\pi}{q}\frac{dq}{d\pi} = \frac{P}{D}\frac{dD}{dP} = \frac{\frac{14000}{23}}{\frac{3900}{23}} * -\frac{1}{20} = -\frac{7}{39} \approx -0.18$$

Since this value is between -1 and $+1$, this would be called *inelastic*.

2.4 Calculate the corresponding consumers' gross benefit and surplus.

The consumers' gross benefit and surplus is easiest to demonstrate as areas on the plot in Figure 2. Each area is the product of the quantity $[MWh]$ and the price $[\$/MWh]$ which results in dollars [$].

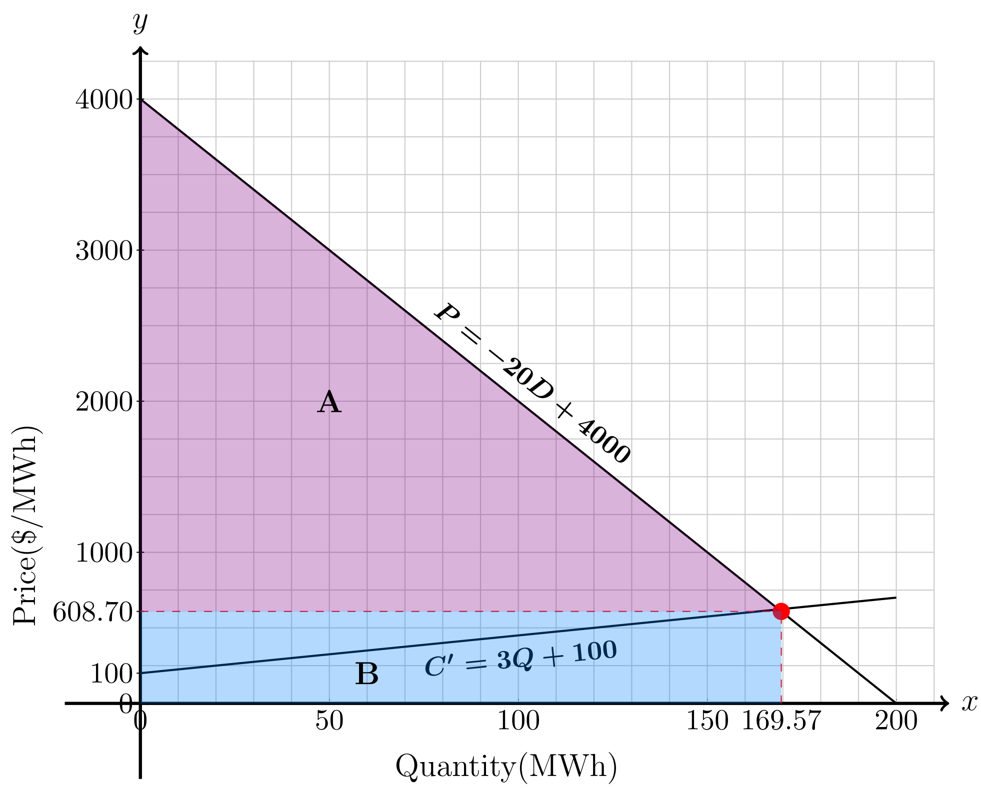

Figure 2: The plot from Figure 1 shading regions to show consumers' gross benefit (A+B) and surplus (A).

Consumers' Net Surplus = A:

$$\frac{1}{2}\left(4000 - \frac{14000}{23}\right)\left(\frac{3900}{23}\right) = \frac{152100000}{529} \approx \$287,523.63$$

Consumers' Gross Surplus = Consumers' Net Surplus (A) + Total Paid (B):

$$\frac{152100000}{529} + \frac{14000}{23}\frac{3900}{23} = \frac{206700000}{529} \approx \$390,737.24$$

Note Consumers' Gross Surplus is also easily found through integration:

$$\int_0^{\frac{3900}{23}} -20x + 4000dx = \frac{206700000}{529} \approx \$390,737.24$$

2.5 Calculate the corresponding producers revenue and economic profit.

The producers gross revenue and economic profit are easiest to demonstrate as areas on the plot in Figure 3. Each area is the product of the quantity $[MWh]$ and the price $[\$/MWh]$ which results in dollars [$].

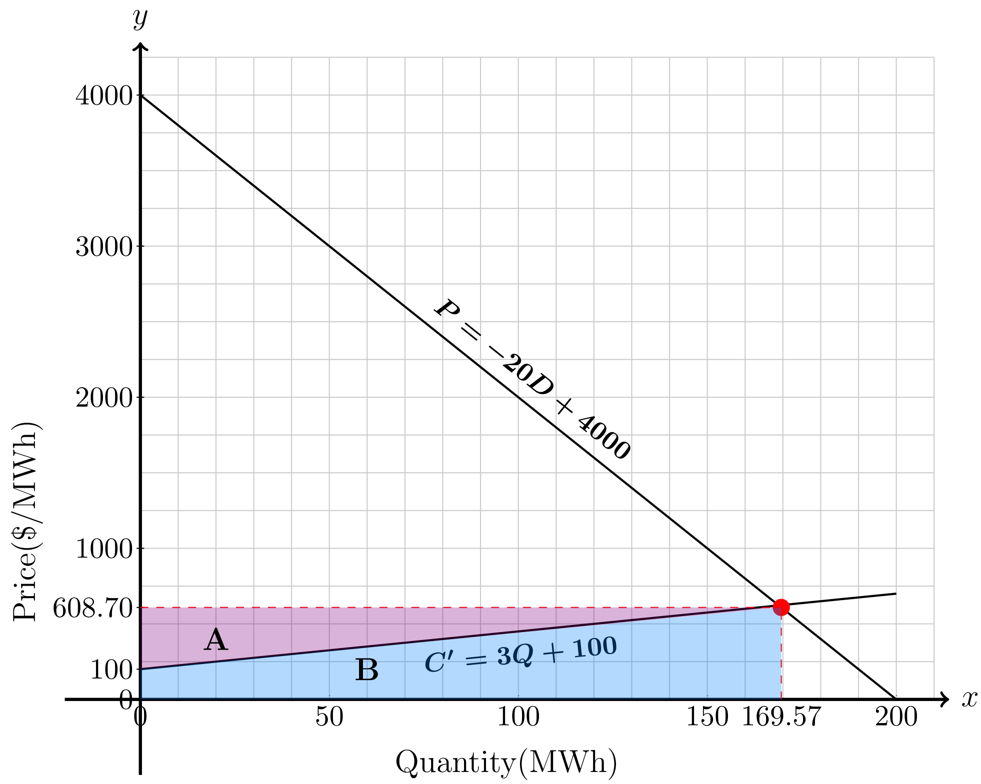

Figure 3: The plot from Figure 1 shading regions to show producers gross revenue (A+B) and economic profit (A).

Producers Gross Surplus (Revenue) = Total Paid = $A + B$ = Price * Quantity:

$$\frac{3900}{23}\frac{14000}{23} = \frac{54600000}{529} \approx \$103,213.61$$

Producers Net Surplus (Profit) = A, which is Revenue - Cost:

$$\frac{54600000}{529} - (1.5Q^2 + 100Q) = \frac{54600000}{529} - \left(\frac{3}{2}\left(\frac{3900}{23}\right)^2 + 100\frac{3900}{23}\right) = \frac{22815000}{529} \approx \$43,128.54$$

2.6 The Syldavian government is considering imposing a tax of \$20.00 per MWh produced on generators. Calculate how much revenue tis tax would raise during the hour of peak demand and the effect that it would have on the global welfare.

When the producer is taxed, this increases their cost, cutting into their profit and effectively reducing the price. This produces a new supply function:

$$C' = 3Q + 120$$

The 120 represent the original $100\$/MWh$ plus the $20\$/MWh$ tax cost. Solving for the new equilibrium price:

$$C' = 3Q + 120 \qquad\qquad P = -20D + 4000$$

$$\begin{aligned} 3Q + 120 &= -20D + 4000 \\ 23Q &= 3880 \\ \therefore Q = D &= \frac{3880}{23} \approx 168.70MWh \end{aligned}$$

At a quantity of $168.70MWh$, the price is calculated:

$$3\left(\frac{3880}{23}\right) + 120 = -20\left(\frac{3880}{23}\right) + 4000 = \frac{14400}{23} \approx \$626.09/MWh$$

The effect of this on global welfare is difficult to demonstrate with our existing plots because the differences are so minute in comparison to the maximum values. To better show what is happening, the important data points are represented in Figure 4 but drawn way out of proportion to exaggerate the shape.

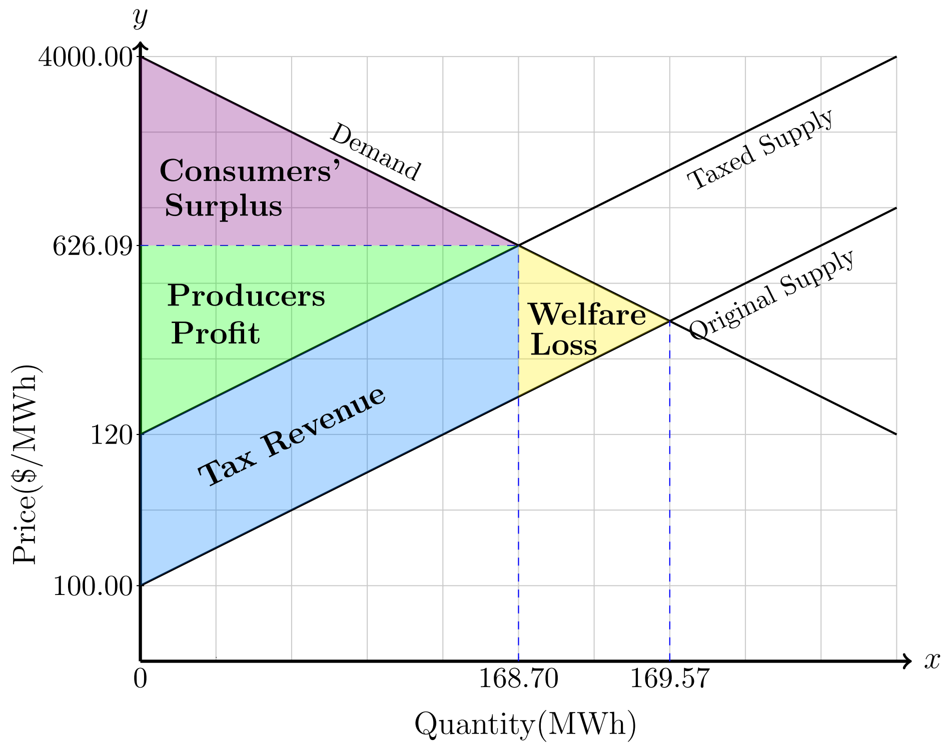

Figure 4: Taxing producers lowers their received price which causes them to lower their supply, effecting global welfare. Note: not to scale.

Using the plot in Figure 4 and the following known functions, we can calculate the dollar value of each region.

$$\begin{aligned}\text{Demand: } P &= -20D + 4000\\ \text{Supply: } C' &= 3Q + 120\\ \text{Equilibrium: } D = Q &= \frac{3880}{23} \approx 168.70\\ \text{Equilibrium: } P = C' &= \frac{14400}{23} \approx 626.09\end{aligned}$$

- Consumers' Surplus (CS): $\frac{1}{2}\left(4000 - \frac{14400}{23}\right)\frac{3880}{23} = \frac{150544000}{529} \approx \$284,582.23$
- Producers Profit (PP): $\frac{1}{2}\left(\frac{14400}{23} - 120\right)\frac{3880}{23} = \frac{22581600}{529} \approx \$42,687.33$
- Tax Revenue (TR): $20 * \frac{3880}{23} = \frac{77600}{23} \approx \$3,373.91$
- Global Welfare: $CS + PP + TR = \frac{150544000}{529} + \frac{22581600}{529} + \frac{77600}{23} = \frac{7604800}{23} \approx \$330,643.48$

The original global welfare is calculated using the results from 2.4 (original consumer surplus) and 2.5 (original producer profit). Then, the Global Welfare listed above is subtracted, providing the Welfare Loss:

$$\frac{152100000}{529} + \frac{22815000}{529} - \frac{7604800}{23} = \frac{200}{23} \approx \$8.70$$

2.7 The Syldavian government is considering imposing a tax of \$20.00 per MWh purchased by customers. Compared to (6), what difference does it make?

When the consumer is taxed, the resulting calculations are similar to those for the producer except that the demand curve is going to shift down. This is because consumers will still only pay a maximum of $4000\$/MWh$ but now they are taxed $20\$/MWh$. This produces a new demand function:

$$P = -20D + 3880$$

The 3880 represents the original $4000\$/MWh$ minus the $20\$/MWh$ tax cost. Solving for the new equilibrium price:

$$C' = 3Q + 100 \qquad\qquad P = -20D + 3980$$

$$\begin{aligned} 3Q + 100 &= -20D + 3980 \\ 23Q &= 3880 \\ \therefore Q = D &= \frac{3880}{23} \approx 168.70MWh \end{aligned}$$

At a quantity of $168.70MWh$, the price is calculated:

$$3\left(\frac{3880}{23}\right) + 100 = -20\left(\frac{3880}{23}\right) + 3980 = \frac{13940}{23} \approx \$606.09/MWh$$

Relying on another plot, we can see the shift of the demand curve in Figure 5.

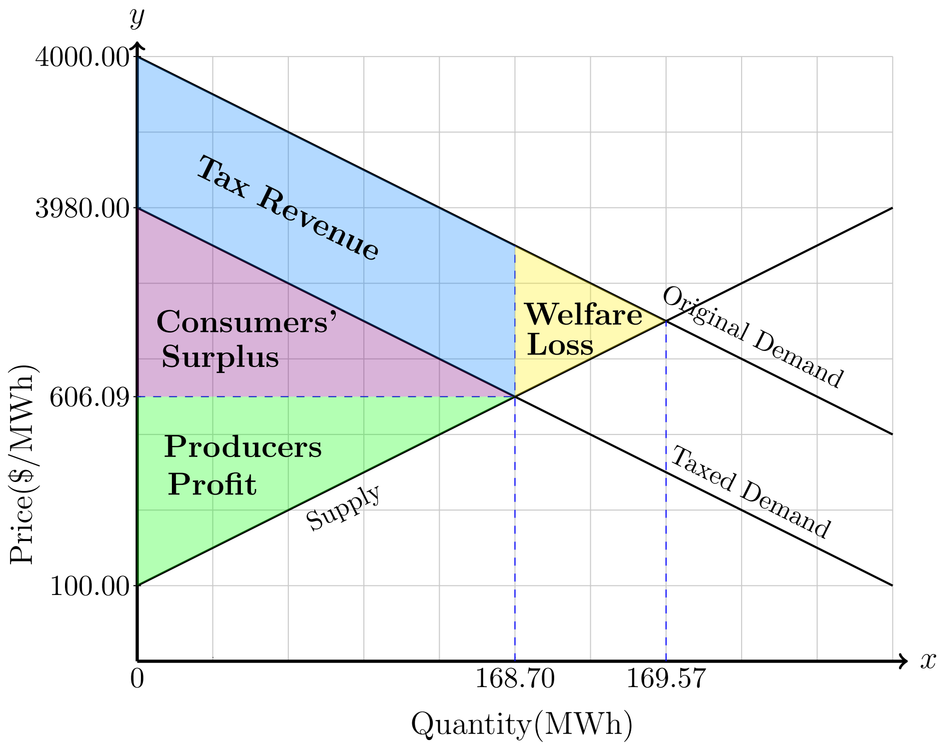

Figure 5: Taxing consumers lowers their maximum they are willing to pay, effecting global welfare. Note: not to scale.

Using the plot in Figure 5 and the following known functions, we can calculate the dollar value of each region.

$$\begin{aligned}
\text{Demand: } P &= -20D + 3980 \\
\text{Supply: } C' &= 3Q + 100 \\
\text{Equilibrium: } D = Q &= \frac{3880}{23} \approx 168.70 \\
\text{Equilibrium: } P = C' &= \frac{13940}{23} \approx 606.09
\end{aligned}$$

- Tax Revenue (TR): $20 * \frac{3880}{23} = \frac{77600}{23} \approx \$3,373.91$
- Consumers' Surplus (CS): $\frac{1}{2}\left(3980 - \frac{13940}{23}\right)\frac{3880}{23} = \frac{150544000}{529} \approx \$284,582.23$
- Producers Profit (PP): $\frac{1}{2}\left(\frac{13940}{23} - 100\right)\frac{3880}{23} = \frac{22581600}{529} \approx \$42,687.33$
- Global Welfare: $(TR) + (CS) + (PP) = \frac{77600}{23} + \frac{150544000}{529} + \frac{22581600}{529} = \frac{7604800}{23} \approx \$330,643.48$

The welfare loss is calculated similarly to 2.6 and it turns out to be the same. So who we tax doesn't matter. If I was the producer, I might want the consumer taxed and vice verse.

$$\frac{152100000}{529} + \frac{22815000}{529} - \frac{7604800}{23} = \frac{200}{23} \approx \$8.70$$

2.8 Currently there is only one generation company in Syldavia. What would be the welfare loss? Hint: this company will also try to maximize its own profit, but unlike Slide 36 in the lecture, the price will depend on the generation output via demand curve.

If the sole generation company in Syldavia wants to maximize profit, the first thing we need is function for profit.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Revenue Revenue can be calculated by multiplying the consumer demand price function by the power supply level:

$$\text{Revenue} = (-20Q + 4000) * Q$$

We already have a cost function:

$$\text{Cost} = 1.5Q^2 + 100Q$$

Putting these together:

$$\begin{aligned}
\text{Profit} &= (\text{Revenue}) - (\text{Cost}) \\
&= [(-20Q + 4000) * Q] - (1.5Q^2 + 100Q) \\
&= -20Q^2 + 4000Q - 1.5Q^2 - 100Q \\
&= -21.5Q^2 + 3900Q
\end{aligned}$$

To find the maximum of this equation, we take the derivative and set it equal to zero:

$$\begin{aligned}
\frac{dP}{dQ} &\equiv -43Q + 3900 = 0 \\
43Q &= 3900 \\
\therefore Q &= \frac{3900}{43} \approx 90.70MWh
\end{aligned}$$

Since the second derivative is negative, this verifies we have a local maximum:

$$\frac{d^2P}{dQ^2} = -43$$

Solving for the price and profit:

$$\begin{aligned}
\text{Profit:} \quad & -21.5 * \left(\frac{3900}{43}\right)^2 + 3900 * \left(\frac{3900}{43}\right) = \frac{7605000}{43} \approx \$176,860.47 \\
\text{Price:} \quad & -20 * \frac{3900}{43} + 4000 = \frac{94000}{43} \approx \$2186.05/MWh
\end{aligned}$$

This would be like setting a price floor of $\$2186.05/MWh$ which is crazy. Calculating the welfare for this, using another plot is useful as shown in Figure 6.

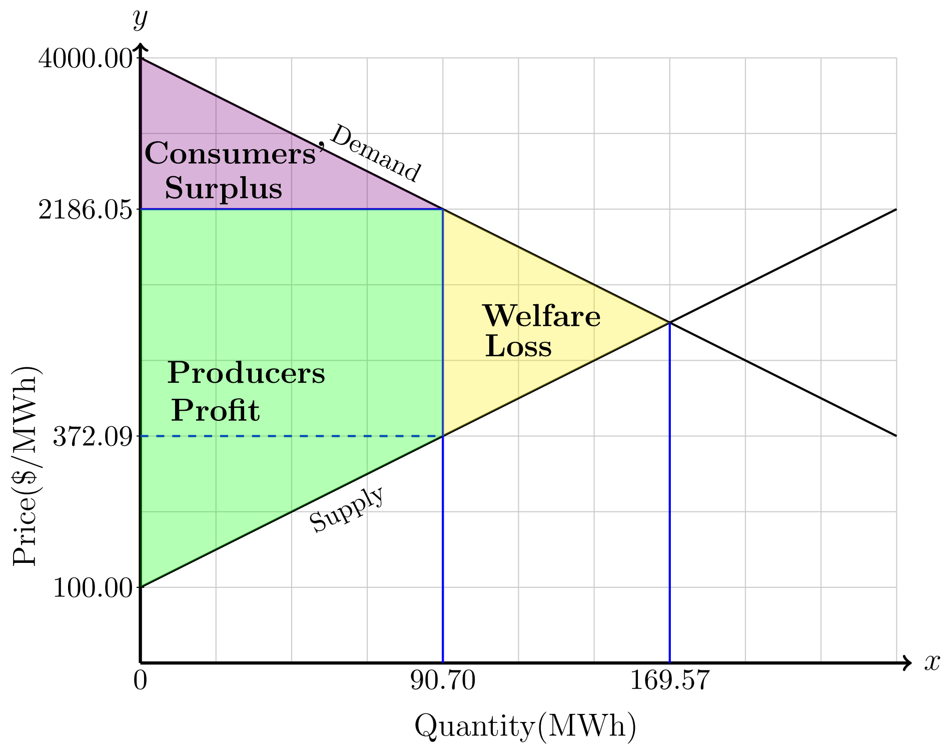

Figure 6: A plot showing consumer surplus, producer profit, and welfare when the power producer sets the price at $\$2050/MWh$ in order to maximize their profit. Note: not to scale.

Running the numbers for the plot in Figure 6:

- Consumers' Surplus (CS): $\frac{1}{2}\left(4000 - \frac{94000}{43}\right) * \frac{3900}{43} = \frac{152100000}{1849} \approx \$82,260.68$
- Producers Profit (PP): $\frac{94000}{43} * \frac{3900}{43} - \left(\frac{3}{2} * \left(\frac{3900}{43}\right)^2 + 100 * \frac{3900}{43}\right) = \frac{7605000}{43} \approx \$176,860.47$
- Global Welfare: $CS + PP = \frac{152100000}{1849} + \frac{7605000}{43} = \frac{479115000}{1849} \approx \$259,121.15$
- Welfare Loss: $\frac{1}{2} * \left(\frac{3900}{23} - \frac{3900}{43}\right) * \left(\frac{94000}{43} - \frac{16000}{43}\right) = \frac{3042000000}{42527} \approx \$71,531.03$

This is a crazy amount of welfare loss. In my opinion, this helps show why allowing a power company to police themselves is a bad idea. It actually demonstrates why allowing any free market to run unregulated is probably a bad idea. This is because almost every free market tends towards monopolies and monopolies, maximizing for profit, will do what we see here.

References

[1] Daniel S. Kirschen and Goran Strbac. *Fundamentals of Power System Economics*. 1st ed. Wiley, May 2004. ISBN: 9780470845721. URL: http://amazon.com/o/ASIN/0470845724/.