Homework 5

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Chapter 9

Exercise 14

A type 2 wind turbine has blades of 50 m length. The induction generator of the turbine is a six-pole machine. The inductive reactance of the core is 3.0 Ω. The rotor resistance and inductive reactance referred to the stator are 0.01 Ω and 0.1 Ω, respectively. The stator winding impedances are much smaller than the core inductive reactance. Because of an increase in wind speed, the generator speed increases from 1260 to 1290 rpm. Compute the added resistance to the rotor circuit to maintain the lift force constant.

Known values:

$$\begin{aligned} r_c &= 50 \text{ m} \\ f &= 60 \text{ Hz} \\ p &= 6 \\ n_1 &= 1260 \text{ rpm} \\ n_2 &= 1290 \text{ rpm} \\ x_m &= 3\ \Omega \\ r_2' &= 0.01\ \Omega \\ x_2' &= 0.1\ \Omega \end{aligned}$$

First a few details. These assumptions are made in the book so I will use its form to solve this problem. We ignore the stator winding impedances. When we then reduce the left side of the circuit in Figure 1, $V_{th} \approx V_{a1}$ and $Z_{th} = 0$ since the shorted V_{a1} and assumption of $r_1 + x_1 = 0$ in parallel with x_m is 0. The solution for this problem is worked using the following steps:

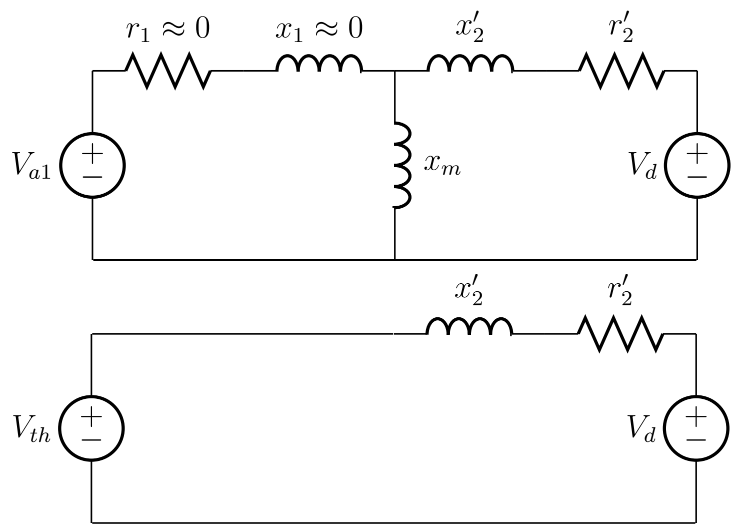

Figure 1: A circuit representing the turbine (top) and its reduced form (bottom).

1. Solve for n_s with the given values.

$$n_s = 120 \cdot \frac{f}{p} = 120 \cdot \frac{60}{6} = 1200 \text{ rpm}$$

2. Use the provided n_1 value to calculate the slip s_1 and mechanical rotor speed ω_{rm}.

$$s_1 = \frac{n_s - n_1}{n_s} = \frac{1200 - 1260}{1200} = -0.05$$
$$\omega_{rm1} = \frac{2\pi}{60} \cdot 1260 \text{ rad/s}$$

3. Use the relationships between (a) torque and force lift constant F_L and (b) torque and angular velocity to calculate the P_{d1} to P_{d2} ratio.

$$T_1 = F_L r_c$$
$$P_{d1} = T_1 \omega_{rm1}$$
$$\therefore \frac{P_{d1}}{\omega_{rm1}} = F_L r_c$$

Since we know that we must maintain the force lift constant F_L:

$$T_2 = F_L r_c$$
$$P_{d2} = T_2 \omega_{rm2}$$
$$\therefore \frac{P_{d2}}{\omega_{rm2}} = F_L r_c$$
$$\therefore \frac{P_{d1}}{\omega_{rm1}} = \frac{P_{d2}}{\omega_{rm2}}$$
$$\therefore P_{d2} = \frac{\omega_{rm2}}{\omega_{rm1}} \cdot P_{d1} = \frac{s_2}{s_1} \cdot P_{d1}$$
$$\therefore P_{d2} = \frac{1290}{1260} \cdot P_{d1}$$

4. Use the equation for power to solve for P_{d1}. Since V_{a1} is needed for the equation but not given and we know it does not change, we can put any value in or just eliminate it.

$$P_{d-max} = 3V_D I'_{a2}$$
$$V_D = \left(-\frac{r'_2 + r'_{\text{add}}}{s}(1-s)\right) \cdot I'_{a2}$$
$$\therefore P_{d-max} = -3 \cdot \frac{1-s}{s} \cdot \frac{V_{th}^2 (r'_2 + r'_{\text{add}})}{\left(r_{th} + \left(\frac{r'_2 + r'_{\text{add}}}{s}\right)\right)^2 + x_{eq}^2}$$

(full derivation provided by Ahmad and attached)

$$\therefore P_{d1} = -3 \cdot \frac{1-s_1}{s_1} \cdot \frac{V_{a1}^2 (r'_2 + r'_{\text{add}})}{\left(\frac{r'_2 + r'_{\text{add}}}{s_1}\right)^2 + x_2^2}$$
$$= -3 \cdot \frac{1-(-0.05)}{-0.05} \cdot \frac{V_{a1}^2 (0.01+0)}{\left(\frac{0.01+0}{-0.05}\right)^2 + 0.1^2}$$
$$\therefore P_{d1} = 12.6 \cdot V_{a1}^2$$

5. Multiply the result for P_{d1} by the $P_{d1} : P_{d2}$ ratio to approximate P_{d2}.

$$P_{d2} = \frac{1290}{1260} \cdot P_{d1}$$
$$= \frac{1290}{1260} \cdot (12.6 \cdot V_{a1}^2)$$

6. Use the provided n_2 value to calculate the slip s_2.

$$s_2 = \frac{n_s - n_2}{n_s} = \frac{1200 - 1290}{1200} = -0.075$$
$$\omega_{rm2} = \frac{2\pi}{60} \cdot 1290 \text{ rad/s}$$

7. Use the equation for power with P_{d2} and s_2 to solve for r_{add}. Notice how the V_{a1}^2 terms cancel after the first line.

$$12.6 \cdot \frac{1290}{1260} \cdot V_{a1}^2 = -3 \cdot \frac{1 - s_2}{s_2} \cdot \frac{V_{a1}^2 (r_2' + r_{\text{add}}')}{\left(\frac{r_2' + r_{\text{add}}'}{s_2}\right)^2 + x_2^2}$$
$$\therefore 12.6 \cdot \frac{1290}{1260} = -3 \cdot \frac{1 - (-0.075)}{-0.075} \cdot \frac{(0.01 + r_{\text{add}}')}{\left(\frac{0.01 + r_{\text{add}}'}{-0.075}\right)^2 + 0.1^2}$$

$\rightarrow$ solved using TI-89

$$\boldsymbol{r_{\text{add}} = 0.005\ \Omega}$$

Exercise 15

A type 2 wind turbine has a six-pole, 60 Hz three-phase, Y-connected induction generator. The terminal voltage of the generator is 690 V. The rotor parameters of the machine are $r_2' = 0.01\ \Omega$; $x_2' = 0.1\ \Omega$; and $N_1/N_2 = 2$.

a. Ignore the impedance of the stator winding and compute the speed at maximum torque.

Known values:

$$\begin{aligned} p &= 6 \text{ poles} \\ f &= 60 \text{ Hz} \\ r_2' &= 0.01\ \Omega \\ x_2' &= 0.1\ \Omega \\ V_{a1} &= 690 \text{ V} \\ \therefore n_s &= 120 \cdot \frac{f}{p} = 120 \cdot \frac{60}{6} = 1200 \text{ rpm} \end{aligned}$$

Derivation of maximum torque equation:

$$\begin{aligned} T_d &= \frac{P_d}{\omega_2} = \frac{P_d}{\omega_s(1-s)} \\ &= -\frac{3}{s\omega_s} \frac{V_{th}^2(r_2' + r_{\text{add}}')}{\left(r_{th} + \left(\frac{r_2'+r_{\text{add}}'}{s}\right)\right)^2 + x_{eq}^2} \\ &\qquad \textit{derive with resepect to s, set equal to 0, and solve for s} \\ s^* &= -\frac{r_2' + r_{\text{add}}'}{\sqrt{r_{th}^2 + x_{eq}^2}} \end{aligned}$$

Applying our knowns about a simplified model where $V_{th} \approx V_{a1}$ and $Z_{th} \approx 0$, slip at maximum torque is:

$$\begin{aligned} s^* &= -\frac{r_2' + r_{\text{add}}'}{\sqrt{r_{th}^2 + x_{eq}^2}} \\ &= -\frac{r_2' + r_{\text{add}}'}{x_2'} \end{aligned}$$

When $r_{\text{add}}' = 0$, the speed at maximum torque is:

$$\begin{aligned} s^* &= -\frac{r_2'}{x_2'} \\ &= -\frac{0.01}{0.1} \\ &= -0.1 \end{aligned}$$

Calculating the speed at a slip of -0.1:

$$\begin{aligned} -0.1 &= \frac{n_s - n^*}{n_s} = \frac{1200 - n^*}{1200} \\ \therefore n^* &= 1320 \text{ rpm} \end{aligned}$$

b. Assume a solid-state switching circuit is used to regulate 0.05 Ω resistance in the rotor circuit. Compute the duty ratio of the switching circuit that makes the maximum torque occur at 10% higher speed.

Steps to solve this problem:

1. Calculate the new desired maximum torque speed n^*. A 10% higher speed means we need:

$$n^* = 1.1 \cdot 1320 = 1452 \text{ rpm}$$

2. Calculate the slip at this speed:

$$\begin{aligned} s &= \frac{n_s - n}{n_s} = \frac{1200 - 1452}{1200} \\ &= -0.21 \end{aligned}$$

3. Use the slip at maximum torque equation to solve for r'_{add} which is the needed resistance referred to the stator:

$$\begin{aligned} s^* &= -\frac{r'_2 + r'_{\text{add}}}{x'_2} \\ \therefore -0.21 &= \frac{0.01 + r'_{\text{add}}}{0.1} \\ \therefore r'_{\text{add}} &= 0.011 \ \Omega \end{aligned}$$

4. Convert the added resistance seen from the stator r'_{add} to the resistance from the rotor side. This is usually not necessary because we have never been given a winding ratio different than 1:1. But in this case we have $N_1/N_2 = 2$.

$$\begin{aligned} \frac{r'_{\text{add}}}{r_{\text{add}}} &= \left(\frac{N_1}{N_2}\right)^2 \\ \frac{0.011}{r_{\text{add}}} &= (2)^2 \\ \therefore r_{\text{add}} &= 0.00275 \ \Omega \end{aligned}$$

5. Using the known resistance of $r = 0.05 \ \Omega$ and the function for r_{add}, solve for the duty ratio k:

$$\begin{aligned} r_{\text{add}} &= \frac{1}{\tau}\int_{t_{\text{on}}}^{\tau} r dt = \frac{r(\tau - t_{\text{on}})}{\tau} = r \cdot \left(1 - \frac{t_{\text{on}}}{\tau}\right) \\ k &= \frac{t_{\text{on}}}{\tau} \\ \therefore r_{\text{add}} &= r(1-k) \\ 0.00275 &= 0.05(1-k) \\ \therefore \boldsymbol{k} &= \mathbf{0.945} \end{aligned}$$

Exercise 17

A 1.5 MVA, type 2 wind turbine has an eight-pole, 60 Hz three-phase, Y-connected induction generator. The terminal voltage of the generator is 690 V. When the machine was running at 936 rpm, the output power was 1.0 MW. After a voltage depression on the grid, the output power of the generator is reduced by 30%. Compute the value of the added resistance that would limit the rotor current to 150% of the rated current of the stator. You may ignore the losses of the generator.

Known values:

$$\begin{aligned} p &= 8 \text{ poles} \\ f &= 60 \text{ Hz} \\ \therefore n_s &= 120 \cdot \frac{f}{p} = 120 \cdot \frac{60}{8} = 900 \text{ rpm} \\ n &= 936 \text{ rpm} \\ V_{a1} &= 690 \text{ V} \\ P_{d1} &= 1.0 \text{ MW} \\ P_{d2} &= 0.7 \text{ MW} \end{aligned}$$

1. Calculate the slip at the beginning of the fault:

$$\begin{aligned} s &= \frac{n_s - n}{n_s} = \frac{900 - 936}{900} \\ &= -0.04 \end{aligned}$$

2. Using the provided rated power of 1.5 MVA, we can calculate the rated current I_{a1}:

$$\begin{aligned} \bar{S}_{a1} &= 3\bar{V}_{a1}\bar{I}_{a1}^* \\ 1500000 &= 3 \cdot \frac{690}{\sqrt{3}} \cdot \bar{I}_{a1}^* \\ \therefore |I_{a1}| &= 1255.11 \text{ A} \end{aligned}$$

3. Since we are ignoring generator losses, that means that our stator and motor currents will be the same.

$$I'_{a2} \approx I_{a1} = 1255.11 \text{ A}$$

4. Calculate the target current to limit the rotor to:

$$I'_{a2} = 1.5 \cdot 1255.11 = 1882.66 \text{ A}$$

5. Calculate the power the resistor will need to dissipate:

$$\begin{aligned} P_{\text{add}} &= P_{d1} - P_{d2} \\ &= 300 \text{ kW} \end{aligned}$$

6. Calculate the desired r_{add} per phase. The provided power was given for the three-phase generator so our $P_{\text{add}} = 100$ kW:

$$\begin{aligned}
P_{\text{add}} &= r'_{\text{add}} \cdot (I'_{a2})^2 \\
\therefore r_{\text{add}} &= \frac{P_{\text{add}}}{(I'_{a2})^2} \\
&= \frac{300000/3}{(1866.66)^2} \\
\therefore \boldsymbol{r_{\text{add}}} &= \mathbf{0.0282 \ \ \Omega}
\end{aligned}$$

Appendix A Attachments

- *Ahmad-PdDerivation.pdf* - Detailed derivation of the P_d equation from Chapter 9 by Ahmad Milyani.

$$I'_{a2} = \frac{-V_{th}}{r_{eq} + jx_{eq}}$$

$$= \frac{-V_{th}}{r_{eq} + jx_{eq}} \times \frac{r_{eq} - jx_{eq}}{r_{eq} - jx_{eq}}$$

$$= \frac{-V_{th}\left(r_{eq} - jx_{eq}\right)}{r_{eq}^2 + x_{eq}^2}$$

$$P_d = 3V_D\left(I'_{a2}\right)^*$$

since V_D is a proportional to I'_{a2} for a given slip, the calculation for $I'_{a2}\left(I'_{a2}\right)^*$ is:

$$I'_{a2}\left(I'_{a2}\right)^* = \frac{-V_{th}\left(r_{eq} - jx_{eq}\right)}{r_{eq}^2 + x_{eq}^2} \times \frac{-V_{th}\left(r_{eq} + jx_{eq}\right)}{r_{eq}^2 + x_{eq}^2}$$

$$= \frac{V_{th}^2}{\left(r_{eq}^2 + x_{eq}^2\right)^2} \times \left(r_{eq}^2 + x_{eq}^2\right)$$

$$= \frac{V_{th}^2}{r_{eq}^2 + x_{eq}^2}$$

Note: It should be $|I'_{a2}|^2$ in equation (9.20)